IN THE UNITED STATES DISTRICT COURT
FOR THE MIDDLE DISTRICT OF NORTH CAROLINA

LEAGUE OF WOMEN VOTERS OF NORTH CAROLINA, et al.,	
Plaintiffs,	Civil Action No. 1:13-CV-660
v.	**RULE 26(A)(2)(B) EXPERT REPORT AND DECLARATION OF THEODORE T. ALLEN, PhD**
THE STATE OF NORTH CAROLINA, et al.,	
Defendants.	

I.   INTRODUCTION

1.   I have been retained by Plaintiffs' Counsel as an expert witness in the above-captioned case. Plaintiffs' Counsel requested that I offer my opinions as to: (1) whether HB 589, if it had been in effect in the most recent general election in North Carolina, would have caused longer lines at polling places and longer average waiting times to vote; and (2) the possible effect of such waiting times, if any, on voter turnout. As explained below, I conclude that eliminating seven days of early voting before the 2012 election would have caused waiting times to vote on Election Day in North Carolina to increase substantially, from a low-end estimate of an average of 27 minutes, to a worst-case scenario of an average of 180 minutes of waiting. Moreover, I further conclude that, as a result of longer lines, a significant number of voters would have been deterred from voting on Election Day in 2012 (with a conservative estimate of several thousand). Finally, I conclude that, barring some additional changes to the law or to the resources allocated to polling places, HB 589's reductions to early voting are likely to result in longer average waiting times to vote in future elections.

II. QUALIFICATIONS

2.   I have been an associate professor of Industrial Engineering in the Industrial Systems & Engineering Department at The Ohio State University (OSU) since 2004. I received my Bachelor of Arts degree from Princeton University (Physics), my Masters degree from the University of California at Los Angeles (UCLA, Physics), and my Ph.D. from The University of Michigan (Ann Arbor, Industrial and Operations Engineering). I have been working in industrial and operations engineering—which includes the study of waiting times for service processes— for 20 years, dating back to a full-time internship at The Ford Motor Company. I have attached as Appendix A to this report a copy of my Curriculum Vitae. The following is a brief summary of my education and experience, particularly as it relates to election systems.

3.   I have been working at OSU, first as a lecturer, then as an assistant professor, and then as an associate professor, since 1996. The classes that I teach at OSU have included: Applied Waiting Line Analysis (which directly relates to predicting how lines develop as a function of customer arrival and machine service processes); Statistical Modeling, Queuing, and Lean Production (which addresses the mathematical framework of simulation and queuing); Design of Engineering Experiments (addressing statistical planning of engineering experiments); and Empirical Model Building in Industrial Engineering (addressing utility theory and simulation optimization for system design and optimal data collection to support these activities). I have won multiple teaching awards, including the 2000, 2001, 2010, and 2013 Alpha Pi Mu Outstanding Faculty Awards (the top teaching award granted by Industrial & Systems Engineering undergraduate seniors) and the Charles E. MacQuigg Student Award for Outstanding Teaching in 2000.

4.   My research areas of interest include the intersection of statistics, operations research, and engineering. My past and current research projects have received over $1.9 million

in grants, from private corporations and public institutions, such as Ford Motor Co., Honda of America Inc., the Edison Welding Institute, and the Naval Joining Center.

5.   I am the author of over 50 publications, including two textbooks, one of which addresses simulation and waiting line theory applications to election systems. I have produced numerous publications related to studying and mitigating election lines including two recent articles in the *Institute of Industrial Engineers Transactions* (2013) and *Productions and Operations Management* (in press), both of which are respected peer-reviewed journals. Some of my relevant publications related to waiting times and elections include:

- Allen, T. T. (2011), *Introduction to Discrete Event Simulation Theory and Agent-Based Modeling: Voting Systems, Health Care, Military, and Manufacturing*, London: Springer-Verlag. (Textbook teaches discrete event simulation, using election system examples to illustrate all the methods.)

- Yang, M., M. J. Fry, W. D. Kelton, and T. T. Allen (in press), "Improving Voting Systems through Service-Operations Management," *Production and Operations Management*. (Explains multiple approaches for election officials to reduce lines with minimal additional expenses, including using our modeling and optimization approaches for deciding how many machines are needed and allocating them to locations accounting for variable ballot lengths.)

- Yang, M., T. T. Allen, M. J. Fry, and W. D. Kelton (2013), "The Call for Equity: Simulation-Optimization Models to Minimize the Range of Waiting Times," *IIE Transactions* 45(7): 781-795. (Focuses on the simulation optimization methods for allocating voting machines to locations to minimize the expected range in waiting times from longest to shortest.)

- Li, J., T. T. Allen, and K. Akah (2013), "Could Simulation Optimization Have Prevented 2012 Central Florida Election Lines?" in *Proceedings of the 2013 Winter Simulation Conference*, R. Pasupathy, S.-H. Kim, A. Tolk, R. Hill, and M. E. Kuhl, eds. (Attempts to recreate the 2012 central Florida general election via simulation and examine what might have happened if voting booths had been apportioned taking ballot length variation into account.)

- Afful-Dadzie, A., T. T. Allen, A. Raqab, and J. Li (2013), "Sufficiency Model-Action Clarification for Simulation Optimization Applied to an Election System," in *Proceedings of the 2013 Winter Simulation Conference*, R. Pasupathy, S.-H. Kim, A. Tolk, R. Hill, and M. E. Kuhl, eds. (Explores an innovative modeling and visualization approach to optimize taking data limitations into account with an election system hypothetical example.)

- Allen, T.T. and M. Bernshteyn (2006), "Mitigating Voter Wait Times," *Chance: A Magazine of the American Statistical Association* 19(4): 25-34. (Shows how regression methods can produce estimates of the numbers of deterred voters and how queuing theory can provide insights into and validated predictions of poll closing times.)

6. I serve on the editorial boards of several peer-reviewed journals including: the *International Journal of Industrial and Systems Engineering*, the *International Journal of Software and Systems Engineering*, and the *Journal of Experimental Design and Process Optimisation*. I am a fellow of the American Society for Quality (ASQ), and a member of the Institute for Operations Research and the Management Sciences (INFORMS) and the American Statistical Association (AMSTAT) which are all major professional organizations relating to applied statistics and operations research.

7. I also work frequently as a consultant in both the private and public sectors. With respect to elections, I have consulted to help election officials make resource decisions for projects in Franklin County, Ohio (contains Columbus; multiple engagements over years including planning resource allocations for two general elections), Cuyahoga County, Ohio (contains Cleveland; yearlong project including planning resource allocations for one general election), and Onondaga County, New York (contains Syracuse; working for the Board of Elections to evaluate elections readiness). In Franklin County in 2006, I proposed and was the technical leader for the implementation of perhaps the first principled approach for deciding how many voting machines are needed and how they should be allocated in order to maximize efficiency and minimize waiting times to vote here. The elections that I helped plan were uneventful from a waiting line perspective, which was the goal after the disastrously long lines and waiting times to vote in Ohio during the 2004 general election. Since then, I have also served as a member of the Ohio State University Senate Apportionment Ad Hoc Advisory Committee (2010).

8.   Outside of elections, I have served as a consultant on projects at companies that include: LaBarge, Lucent, Nationwide Services, Timken, and Net Jets. My accomplishments have included: leading a team to significantly reduce average discharge times at a community hospital and developing methods to accurately forecast maintenance costs associated with an aging fleet of aircraft.

III.   TESTIMONY AT TRIAL, DEPOSITIONS, AND AFFIDAVITS

9.   I am currently serving as an expert witness in an ongoing matter in New Mexico, concerning whether having insufficient printers at polling sites could have affected the outcome of an election.

IV.   COMPENSATION

10.   My fee for this case is $150 per hour (my normal nonprofit consulting rate), plus reimbursement for personal expenses in connection with the case. My fee for deposition testimony and court testimony at locations other than Ohio is $1,250 per day, plus reimbursement for travel and expenses.

V.   INFORMATION AND DATA CONSIDERED IN FORMING OPINIONS/METHODOLOGY

11.   In preparation for this report I reviewed documents including: the text of HB 589; turnout data from the North Carolina Board of Elections; sample ballots from various counties in North Carolina; information describing the types of voting machines used and the arrival processes for voters at polling places in various counties in North Carolina; and information concerning poll closing times in North Carolina. For a complete list of documents I considered (other than those cited in the text of this report), please see Appendix B.

VI.   BACKGROUND ON WAITING LINE ANALYSIS AND ELECTIONS

12.   The phrases "queuing theory" and "waiting line analysis" refer to an area of practical academic research within the fields of industrial engineering and statistics, related to all operating systems in which there are arrivals, waiting in lines (queuing), and service. Queuing theory is generally accepted in the field of industrial engineering and is frequently employed in a range of industries to predict waiting times and maximize efficiency in a variety of areas, from assembly lines in complex manufacturing processes, to simple consumer queues at coffee shops or amusement parks. Queuing theory can, in many cases of interest, predict the average times people wait, and the relationship between those times and factors such as the properties of the arrival and service processes.

13.   As applied to elections, a queuing model can predict average line length; average waiting times to vote; and the number of voters that a system can handle during a particular time interval. Election systems are relatively simple when compared to other kinds of waiting systems that I have modeled. For example, consider the final assembly of cars at automotive plants, which I have modeled extensively, and which involves hundreds of operations, complicated priority and scheduling inputs, and numerous waiting areas for parts. Election systems, by contrast, are less complicated. People arrive at a polling place at some rate and are served (*i.e.*, check in and vote) at some rate. A line develops if more people arrive than can be serviced in a given period of time.

14.   Generally speaking, line length and waiting times depend on at least three variables: (1) the number of voters arriving at a polling place; (2) the number of "servers" (such as the number of machines on which a voter can cast a ballot); and (3) the "service time," that is, the length of time it takes a voter to cast a ballot (which might be based on factors such as the length of the ballot; the type and complexity of the voting machine; and the availability of

practices that can reduce voting time, such as straight-ticket voting, in which voters make a single marking indicating that they wish to vote for candidates from the same party for all races, rather than marking candidates individually for each race).

15.   Often, lines develop due to a bottleneck occurring at one of the service operations in a system. In my experience, usually only one of the service operations is the bottleneck, and other operations can be left out of modeling without affecting the results. In election systems, the bottleneck is often the part of the operation in which the voter enters votes on a machine, for example, an Election Systems & Software (ES&S) direct recording equipment (DRE) machine (essentially, a touch-screen ATM-style machine). I note that the process in which voters check in or register could also serve as a bottleneck, but for the sake of simplicity, I illustrate below a basic queuing model featuring a single (bottleneck) operation, which can be viewed as a voter "monopolizing" a DRE machine, as shown in Exhibit A.[1] .

Exhibit A. Illustration of a queuing at a voting location with a single bottleneck.

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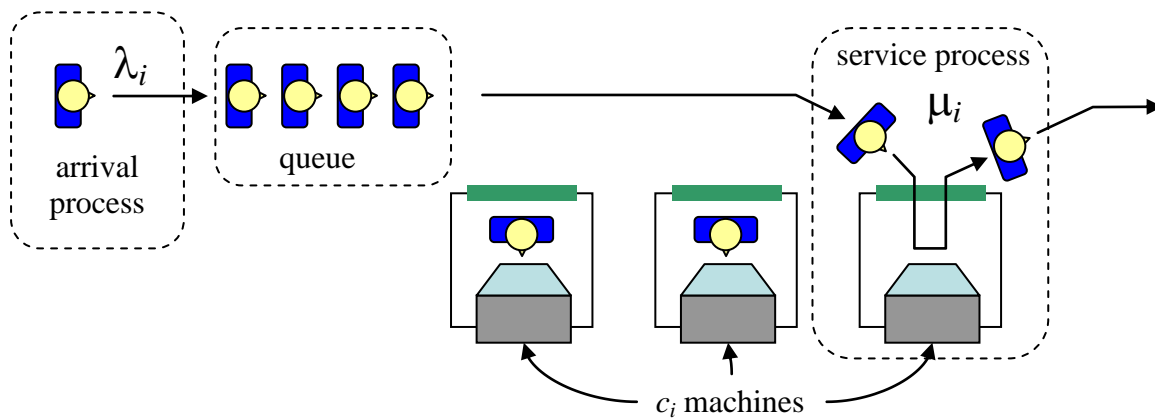
[1] I note that this simple model may be viewed as underestimating the average waiting times; if other services processes like registration or checking in are included, then the total waiting times could only increase.

16.   As described in standard textbooks and in one of my textbooks,[2] a basic queuing system is called the "M/M/c" queue. The parameters as applied to elections are as follows:

λ = the arrival rate, *e.g.*, if 1,300 voters arrive over 13 hours, we have an average arrival rate of 100 voters per hour;

μ = the service rate, *e.g.*, if, when voters are monopolizing a voting machine, they take an average of approximately 4 minutes to vote, we would have a service rate of 15 voters per hour (calculated as 60 minutes ÷ 4 minutes per vote);

c = the number of machines at the relevant location; and

$\rho = \lambda/(\mu c)$ is called the "utilization," or the fraction of the time that servers (here, voting machines) are busy.[3]

17.   Changes to the variables can affect line length and waiting times. For example, if the number of voters arriving during a given time period increases, but other factors (such as the number of voting machines and the time it takes to vote) are held constant, lines may increase, which in turn may increase the waiting time to vote. Similarly, if there are an insufficient number of voting machines, that can affect the length of lines.[4] Or, if the service rate (the average time

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[2] Allen, T. T. (2010). *Introduction to Engineering Statistics and Lean Sigma: Statistical Quality Control and Design of Experiments and Systems*. London: Springer-Verlag. Queuing system described at 83.

[3] Generally speaking, to prevent excessive lines, utilization should never reach 100%. If arrivals were spaced evenly (*i.e.*, voters showed up at regular intervals) and service were perfectly repeatable (voters took exactly the same time with a machine), a system could in theory operate where utilization was at 100%. Given, however, that there will be some randomness in the arrival process, utilization should generally remain below 100%, or there will be what we call an "infinite build-up" in the long run. In other words, if utilization is greater than 100%, the system is in overload, and extremely long lines will eventually develop and grow.

[4] *See, e.g.*, Edelstein, W. A. & Edelstein, A. D. (2010). "Queuing and Elections: Long Lines, DREs and Paper Ballots." *Proceedings of the 2010 international conference on Electronic voting technology/workshop on trustworthy elections*, Article No. 1-8. Berkeley, CA: USENIX Association.

that it takes a voter to cast a ballot) increases (for example, because of ballot length or the type of machine used), that can increase waiting times as well.[5]

18.   Based on the queuing model, the probability of waiting to vote, and expected waiting time, can be estimated using two equations. First, the long run probability that any voter would wait if the arrivals are what we call "extremely random" (that is, uncoordinated with exponentially distributed inter-arrivals, meaning that the time between arrivals is very unpredictable) and the service is also extremely random is:

$$\text{Pr}\{Wait\} = \Pi_W = \frac{(c\rho)^c}{c!}\left((1-\rho)\sum_{n=0}^{c-1}\frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!}\right)^{-1}$$ (Equation 1)

In the above example with $\rho = 0.777$ (meaning that, on average, the voting machines are in use 77.7% of the time), the Pr{*Wait*} = 0.47 or 47% of voters would wait.

19.   Next, the long run probability that any voter would wait (Π_W) can be used in a separate equation to calculate the average expected waiting time:

$$E(W) = \begin{cases} \Pi_W\frac{1}{1-\rho}\frac{1}{c\mu} & for\ \rho < f(h) \\ \frac{(\lambda-c\mu)h}{2c\mu} & for\ \rho > g(h) \end{cases}$$ (Equation 2)

A basic queuing model can generate predictions for average waiting times where utilization is less than 99% (*i.e.*, where $\rho < .99$). Some elections have, unfortunately, operated in an overload condition, where utilization exceeds 99% (the $\rho > 1$ regime), meaning that voters are arriving faster than the servers can handle them, on average. In such scenarios, a basic queuing model predicts lines that are essentially infinite. Obviously, lines cannot stretch into infinity; for one thing, there are a limited number of hours in a day during which the polls are open, such that the line must end at some point. To account for this, in this last equation, I have included an

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[5] *See, e.g.*, Allen, T.T. and M. Bernshteyn (2006), "Mitigating Voter Wait Times," *Chance: A Magazine of the American Statistical Association* 19(4): 25-34.

approximation from a forthcoming peer-reviewed article,[6] using principles of fluid dynamics,

which are generally accepted in my field. This approach enables us to generate predictions for

average waiting times even in some overload situations. In this approximation, h is the number

of hours in an election day, *e.g.*, often $h = 13$ hours. For elections, where utilization is less than

99% ($f(13$ hours$) = 0.99$), it is reasonable to apply Equation 2 with a basic queuing model; and

where utilization is beyond capacity ($g(13$ hours$) = 1.2$), we can apply Equation 2 with a model

employing fluid dynamics. This enables us to arrive at concrete predictions for average waiting

times that continue to increase as the utilization rate (ρ) continues to increase beyond full

capacity.[7]

20.   Before turning to an analysis of North Carolina, I validated (meaning, empirically

tested) Equation 2, using actual elections data, to determine if it would accurately predict the

relationship between factors such as the number of voters arriving and the length of waiting

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[6] *See* Yang, M., M. J. Fry, W. D. Kelton, and T. T. Allen (in press), "Improving Voting Systems through Service-Operations Management," *Production and Operations Management*.

[7] I note that, even using this fluid dynamics approximation, there are a small range of scenarios of overutilization for which there is no reasonably accurate queuing approximation. This is the case where utilization is greater than 99%, but less than 120% (*i.e.*,, where utilization exceeds 1, but $f(h) < \rho < g(h)$). In these cases, simulation is needed to predict average waiting times. By "simulation," I mean stochastic simulation, which refers to a numerical method to estimate complex processes, involving variables that can change randomly. Simulation is one way in which waiting times can be estimated, and which can accommodate a broader set of assumptions than the queuing model that I utilize in this report. In this context, stochastic simulation can attempt to account for random variation in factors such as the number of voters arriving in a given hour or the length of time it takes voters to cast ballots, rather than using average values for those variables. Because stochastic simulation incorporates a degree of randomness, it may produce different outputs even when the starting inputs are the same, requiring many simulations in order to generate a range of probable outcomes. Although I have employed discrete event simulation in my work with election officials, because it is flexible and can model short periods accurately, describing simulation models is much more complicated, such that they are generally less transparent and less able to aid us in generating intuitive insights. Therefore, here, I rely on a non-stochastic "deterministic" queuing theoretic model to predict average waiting times.

times at polling places. I found that Equation 2 predicted with reasonably accuracy the waiting times to vote at 787 precincts in Franklin County, Ohio during the 2004 general election.

21.   In conducting this analysis, my starting point was the waiting times to vote at each of these 787 precincts, so that I could compare the predictions generated by the queuing model to the actual waiting times. Precincts, however, do not generally maintain comprehensive records that record the precise waiting times that voters experience; but they sometimes maintain other forms of data that we can use as a proxy for waiting times to vote at that precinct. In Ohio, for example, precincts maintain information on the time that each polling place records its last vote and closes on Election Day. This can give us a reasonable estimate of waiting times, because no new voters can enter the line at a precinct after the precinct's scheduled closing time, generally 7:00 pm (or 7:30 depending on state rules), but the polls do not actually close until shortly after the last voter has cast his or her ballot. Also, most of the locations were being over-utilized (*i.e.*, in an overload situation) during the election ($\rho > 1$), so the longest lines were expected at the end of the day. Thus, if a precinct must cut off its line at 7:00 pm, but the last ballot at that precinct was not cast until 7:30, we can reasonably infer that the last voter at that polling place waited at least 30 minutes to vote.[8] Thus, the poll closing time at each precinct enables us to determine the length of time that the last voter at that precinct had to wait in order to cast a ballot, which we can then use as an approximation of the waiting times experienced at

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[8] *See* Li, J., T. T. Allen, and K. Akah (2013), "Could Simulation Optimization Have Prevented 2012 Central Florida Election Lines?" in *Proceedings of the 2013 Winter Simulation Conference*, R. Pasupathy, S.-H. Kim, A. Tolk, R. Hill, and M. E. Kuhl, eds.

that polling place. I have used the data from precinct closing times in two of my peer-reviewed articles.[9]

22.   Testing Equation 2, we would expect that, if the model is accurate, it would predict average waiting times at each precinct that would approximate the length of time that the precinct stayed open late. In other words, if the model is accurate, we would expect that it would predict an average waiting time of 15 minutes at a precinct that records its last ballot cast, and actually closes approximately 15 minutes after its scheduled closing time.

23.   Exhibit B shows the predicted waiting time as calculated by Equation 2, plotted against the actual poll closing times for all 787 precincts in Franklin County, Ohio during the 2004 election. Each plotted point represents one of 787 precincts.[10] As we can see, Equation 2 generally predicts longer lines at those precincts that, in fact, had later closing times. The plot shows a strong relationship with predictable variation, given that the prediction is only an average waiting time, and the actual poll closing time is a single value. Thus, Exhibit B indicates that Equation 2 accurately predicts the relationship between factors such as the number of voters arriving at the polls and average waiting times.

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[9] *See* Allen & Bernshteyn (2006), *supra* note 5; Yang, M., T. T. Allen, M. J. Fry, and W. D. Kelton (2013), "The Call for Equity: Simulation-Optimization Models to Minimize the Range of Waiting Times," *IIE Transactions* 45(7): 781-795.

[10] I note that one precinct was omitted because utilization at the precinct exceeded 99% of capacity ($\rho = 0.998$) and, as discussed above, the queuing approximation in Equation 2 cannot generate accurate predictions in that range.

Exhibit B. Predicted Average Waiting Times from Equation 2, and Actual Poll Closing Times, Franklin County 2004 Election.
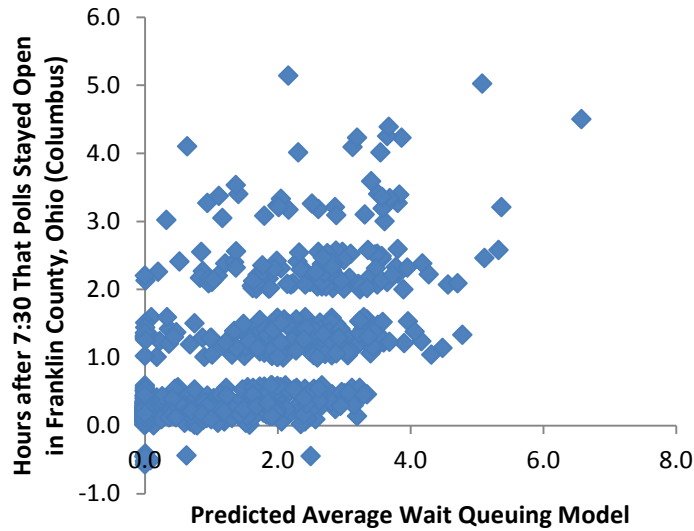
VII.   ANALYSIS OF THE EFFECT OF HB 589'S REDUCTION OF EARLY VOTING DAYS

24.   The questions I have been asked to analyze are: (1) whether HB 589's reductions in the early voting period, had they been in effect during the 2012 general election, would have caused longer lines and waiting times to vote, by shifting voters on to Election Day who might otherwise have voted in-person during the now-eliminated days; and (2) whether that increase in waiting times may have deterred some people from voting. Essentially, the question is whether, in 2012, moving some or all of the voters from the now-eliminated early voting days over to voting on Election Day would have resulted in longer waiting times and the deterrence of some voters.

A. The Effect on Waiting Times to Vote

25.   The starting point for this analysis is the average waiting times of North Carolinians in recent elections. In the most recent general election, in2012, average waiting times to vote in North Carolina were reported as being approximately 13 minutes.[11]

26.   Using Equation 2 then allows us to create a picture of a typical voting location in North Carolina in 2012. If an average polling location has 100 people arriving per hour (in total at the location), a total of 7 voting machines ($c = 7$ servers), and an average time required to be served by the machines of 4.035 minutes,[12] then Equation 2 predicts an average waiting time of 13 minutes (which is consistent with what was reported for North Carolina). Note that this example of an average precinct is meant for illustrative purposes only. The numbers I have

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[11] *See* Stewart III, C. (2013), "Waiting to Vote in 2012," *Journal of Law and Politics* 28(4): 439-463. Based on data from two nationwide surveys conducted after the 2012 election (the Survey of the Performance of American Elections (SPAE) and the Cooperative Congressional Election Study (CCES)), Charles Stewart, III of M.I.T. has reported the average number of minutes that voters reported waiting to vote in-person in each state during the 2012 election. Stewart reports average waiting times in North Carolina at about 13 minutes or 0.217 hours. *See id.* at 450. The Pew Center similarly reports average waiting times to vote in North Carolina during the 2012 elections of approximately 13.5 minutes. *See* Pew Charitable Trusts, The Elections Performance Index 2012, The State of Election Administration and Prospects for the Future, North Carolina, *available* at:
http://www.pewstates.org/uploadedFiles/Flash_Library/PCS/Interactives/ElectionsPerformanceI
ndex/template.html#state-NC.

[12] The ~4 minutes service time is consistent with my inspection of recent ballots used in North Carolina, and my own data collection experience while working as an elections consultant, including a mock election in which Ohio voters used similar ES&S machines to vote on races and issues. As I note below, however, there are reasons to think that this is a conservative estimate for average voting times in North Carolina for purposes of predicting waiting times in future elections, given, for example, HB 589's elimination of straight-ticket voting, which will require all voters to mark candidates for each individual race on a ballot, and thereby likely increase the average time that it takes voters to cast a ballot.

chosen do not affect the results of the queuing model when it is used to predict the effect of adding a certain increment of voters on waiting times to vote.[13]

27.   HB 589 is remarkable because it directly eliminates voting days on which a significant fraction of the electorate (19.8%) voted in 2012. Note that in 2012, the total voting on those days was 899,083 voters, and only 1,719,853 people voted in person on Election Day. Exhibit C below shows the numbers of voters on the early days in the most recent general election with the shaded days being those eliminated by HB 589.

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[13] As long as the ratio between voters and service time remains the same, the precise numbers we use do not change the predictions of the queuing model. That is, if we seek to measure the effect of adding a certain percentage more voters to the system, the queuing equations give the same predictions as long as utilization (the quantity ρ) is the same. To give a simple example: for purposes of the model, the results are the same if the typical precinct has 100 voters per hour with a service time of 4 minutes per voter, or if it has 50 voters per hour with a service time of 8 minutes per voter. In the first scenario, if the model assumes that 100 voters arrive with a service time of 4 minutes per voter, then that would indicate 400 minutes of voting machine utilization (100 voters x 4 minutes per voter). In this example, if the number of voters who arrive increases by 10% (10 voters), it would mean an increase in utilization by 40 minutes of machine time (10 voters x 4 minutes each = 40 minutes). Now, if instead the model assumes that 50 voters arrive per hour with a service time of 8 minutes per voter, the results are the same. Here, the machines would still be in use for 400 minutes (50 voters x 8 minutes per voter). In this scenario, if the number of voters increases by 10% (5 voters), the increase in machine utilization time is the same: 40 minutes (5 voters x 8 minutes per voter = 40 minutes). Thus, the precise numbers we start with in building a picture of a typical precinct do not alter the results when modeling the effect of increased arrivals.

Exhibit C. Ballots cast during early voting in North Carolina during recent general elections.[14]

Day	Voters
Thursday	166,617
Friday	156,664
Saturday	63,950
Sunday	25,534
Monday	155,921
Tuesday	161,595
Wednesday	168,802
Thursday	162,373
Friday	182,897
Saturday	110,708
Sunday	35,964
Monday	188,468
Tuesday	168,824
Wednesday	167,968
Thursday	187,697
Friday	249,318
Saturday	202,928
Total on the Now-Eliminated Days	**899,083**
Total Early Voting	**2,556,228**
Total Election Day	**1,719,853**

28.   Now consider that, had HB 589 been effect during the 2012 election, some portion of the people who voted on the now-eliminated days would have tried to vote on Election Day. For example, if all of the 899,083 people who voted on the eliminated days in 2012 instead tried to vote on Election Day, that would represent a 52.3% increase in the number of voters trying to cast a ballot on that day. Of course, had HB 589 been in effect during the 2012 election, not all of those voters would have tried to cast ballots on Election Day. Some may not have voted at all, possibly due to work or family responsibilities, and would have been effectively disenfranchised by the elimination of early voting opportunities on the days in question. Some

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[14] This data was downloaded from the official online FTP archive of the North Carolina State Board of Elections, ftp://www.app.sboe.state.nc.us/ENRS/absentee11xx02xx2010.zip, and provided to me by Plaintiffs' counsel.

may have cast early votes on the remaining days of the early voting period or voted by mail. But at least some of those 899,083 voters would have tried to vote on Election Day.

29.   Because it is impossible to know precisely how many of the voters from the now-eliminated days would have tried to vote on Election Day, I analyzed many scenarios in which some or all of the 899,083 North Carolina voters who voted on the now-eliminated days might have tried to vote on Election Day, representing net increases in the number of voters on Election Day of up to 52%. That is, I used my queuing model to determine average waiting times for various possible scenarios, assuming a net increase of voters arriving on Election Day of various different increments. None of these scenarios are intended to serve as a precise prediction of the length of the lines on Election Day 2012 had HB 589 been in effect at that time, but rather to illustrate the range of possible outcomes if some of these voters had shifted to Election Day.

30.   If voters had shifted from the now-eliminated days to Election Day in 2012, average waiting times to vote could have increased substantially. For example, if just 3.8% of the 899,083 voters from the now-eliminated days tried to vote on Election Day (which translates into 34,397 voters, or a 2% net increase in the number of voters attempting to vote on Election Day), average waiting times would have more than doubled, from 13 to 27 minutes. If approximately a quarter (26.8%) of the voters from the now-eliminated days tried to vote on Election Day (*i.e.*, 240,779 voters, or an increase of 14% in the number of voters attempting to vote on Election Day)), average waiting times to vote would be approximately one hour. In a worst-case scenario, if all 899,083 voters from the now-eliminated days shifted to Election Day, average waiting times would be just shy of three hours. These three scenarios (along with Election Day numbers) are set forth in Exhibit D below:

Exhibit D. Predictions from Equation 2 for scenarios considered (with arrival rates expressed in terms of voters per hour, and wait times expressed in terms of minutes).

% of Voters From Eliminated Days	Total # of Voters Added to ED	% Increase in ED Voters	Arrival Rate (λ)	Exp[Wait] (Minutes)
0%	0	0%	100.0	13.0
3.8%	34,397	2%	102.0	27.0
26.7%	240,779	14%	114.0	37.1
99%	894,324	52%	152.0	179.5

31.   These results indicate that the potential to create long lines on Election Day is a concern. Note, however, that there are reasons to think that these numbers may be conservative, and may underestimate the effects on lines, for several reasons. First, I have employed conservative estimates as to service times. Initially, my estimated service time of 4 minutes is generally conservative (4 minutes is a relatively short period of time in which to cast a ballot). Moreover, HB 589 eliminates straight ticket voting, which would likely significantly increase the time it takes to cast a ballot. At this time, I have not attempted to account for the resulting longer service times in the estimates above. As noted, however, it is quite clear from both queuing theory and my own empirical research that "[s]mall changes in the average time to vote can cause large changes in the average waiting times."[15] And, as service times increase, the marginal effect of adding additional voters to the system may also be magnified. That is, if the average

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[15] Allen and Bershteyn (2006), *supra* note 5, at 27. For example, service times appear to have been the primary driver of long lines in both 2004 Franklin County, Ohio and 2012 central Florida, where elections officials failed to account for highly variable ballot lengths in allocating resources to voting locations. That is, election officials allocated machines based primarily on how many voters they expected to turn out (the arrival process), but did not consider how long it would take voters to cast a ballot (the service process). I note that variable ballot length does not seem to be a major issue in North Carolina, where ballots appear to be uniformly short in comparison to those used in Ohio or Florida during typical elections. Service times, however, may significantly increase as a result of HB 589, for reasons discussed below.

time to cast a ballot is only 4 minutes, and we add 10 voters, that would add 40 minutes of utilization time to a machine. But if the average time to cast a ballot is 10 minutes, then adding 10 voters to the system would add 100 minutes of utilization time to a machine. Thus, if average services times are longer than four 4 minutes, the effect of adding additional voters to Election Day may be more significant, meaning that the calculations above would underestimate the possible effects that eliminating 7 days of early voting would have on line length and waiting times to vote.

B. The Effect of Waiting Times to Vote on Voter Turnout (The "Two Percent Rule")

32.   A queuing model can provide useful information not only as to how changes in variables (*e.g.*, the allocation of machines, the time it takes to vote, or the number of voters arriving) can affect the length of waiting times, but also about how those changes might ultimately affect turnout. As others have noted, waiting in long lines to vote can, among other things, discourage people from voting.[16] In queuing theory terminology, "balking" refers to cases in which people choose not to seek service because they perceive waiting times as unacceptable before entering the lines. "Reneging" is the practice of foregoing service after entering the lines due to the length of the line. Both balking and reneging are important in voting systems because they correspond to reduced voter turnout, or "deterred" votes.[17]

33.   In my own research, I have found that lines were longest in the locations with the lowest turnout.[18] This may seem like a counter-intuitive result: one might expect that long

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[16] *See, e.g.*, Spencer, D. M. & Markovits, Z. S. (2010), "Long Lines at Polling Stations? Observations from an Election Day Field Study," *Election Law Journal* 9(1): 3-17, at 15-16; Alvarez, R. M., Ansolabehere, S., Berinsky, A., Lenz, G., Stewart III, C. & Hall, T. (2009). *2008 Survey of the Performance of American Elections: Final Report*, at 32-36.

[17] Allen & Bernshteyn (2006), *supra* note 5, at 26.

[18] *See, e.g.*, Allen & Bernshteyn (2006), *supra* note 5, at 29; Li, Allen, & Akah (2013), *supra* note 8, at 1.

waiting times would be a consequence and indicator of high levels of voter turnout. Generally speaking, however, long lines can deter would-be voters and thereby suppress turnout. Thus, one Election Day field study observed a statistically significant positive correlation between line length and probability of reneging by would-be voters.[19] Similarly, one 2008 survey found that as many as 11% of nonvoters indicated that excessive lines were a "major factor" for them in not voting, with another 9% describing lines as a "minor factor" in not voting.[20]

34.   Based on regression modeling of election data, there appears to be a direct relationship between waiting times to vote and voter turnout, which we can refer to as the "Two Percent rule." That is, every additional 60 minutes of waiting time results in a decline in turnout of approximately 2% of the remaining eligible voters (those who are registered and who have not voted early).

35.   This rule was initially derived from an inspection of the results of the 2004 Franklin County, Ohio poll closing times[21]. Exhibit E illustrates the turnout at 787 precincts in Franklin County, Ohio, plotted against the waiting times to vote at each precinct (again, using the poll closing time as a proxy for line length). Each point is, again, a precinct and the overlay is the best fit line which is a linear regression. From Exhibit D below, in the 2004 Franklin County elections, we see that, indeed, the turnout was lower in the locations with the latest poll closing times, declining approximately 2% for each additional hour of waiting time to vote.

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[19] *See* Spencer & Markovits (2010), *supra* note 16, at 15-16.

[20] *See* Alvarez, *et al*. (2009), *supra* note 16, at 36.

[21] This data has been described and used in my previous work. *See* Allen & Bernshteyn (2006), *supra* note 5; Yang, Allen, Fry, & Kelton (2013), *supra* note 9.

Exhibit E. Poll Closing Times (in Hours after Line Cut-Off), and Turnout Percentage, Franklin County, 2004.
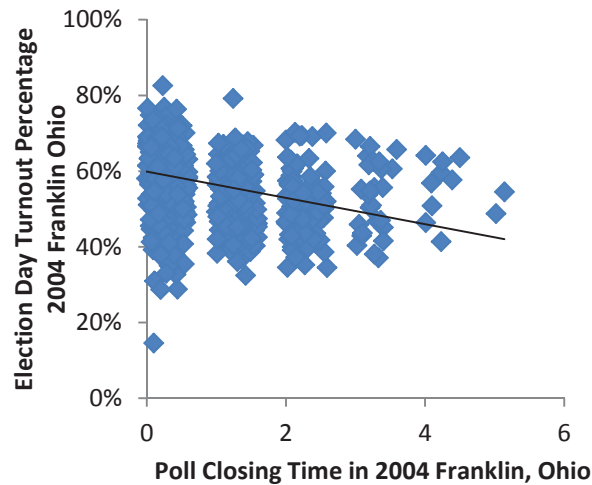
36.   There is no reason to assume that the sensitivity of citizens to waiting lines in elections would differ from place to place. In fact, the fit line provides an approximate rule that has subsequently held well in another recent election in which there were long lines.[22] Exhibit F on the following page illustrates the turnout at central Florida precincts, plotted against the waiting times to vote during the 2012 general election. The fit line in that election is once again approximately consistent with the Two Percent rule. That is, in central Florida during the 2012 election, like in Franklin County, Ohio during the 2004 election, turnout declined by approximately 2% for each additional hour that would-be voters had to wait in line to cast a ballot.

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[22] *See* Li, Allen, & Akah (2013), *supra* note 8; Allen T. T. (2013, Jan. 8), "Delving into the reasons for long lines can bring solutions," *Orlando Sentinel*, *available at* http://articles.orlandosentinel.com/2013-01-08/news/os-ed-long-lines-voting-florida-010813-20130107_1_long-lines-ballot-length-turnout. (Reviews the data included here, showing that ballot length variation drove closing time variation and later closing times were associated with lower turnouts because of deterred voters.)

Exhibit F. Poll Closing Times (in Hours after Line Cut-Off), and Turnout Percentage, Central Florida 2012.
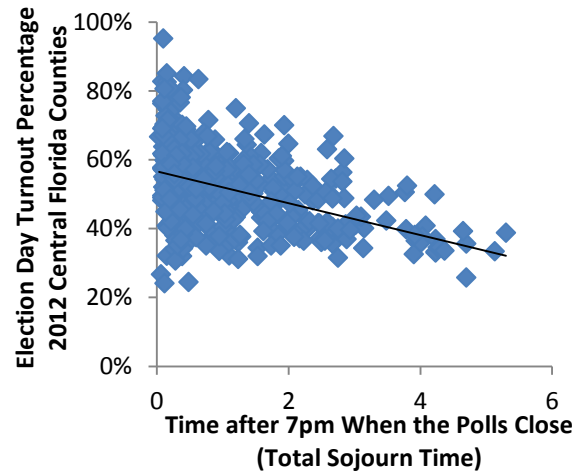
37.   Thus, in order to determine the effect that waiting times to vote can have on turnout, a simple estimate uses the average waiting time in hours, and then multiplies it by 2% of the number of registered voters, excluding those who had voted early. For example, if people are asked to wait 5 hours, typically the turnout will be reduced by 10%. I note that the Two Percent rule may underestimate the total number of voters deterred by waiting lines. Logically, only people aware of the local lines likely would be included in the Two Percent rule, so others could well be deterred because of general awareness including experiences from past elections.

38.   Combining the queuing model predictions of the average waiting times (described above) with the Two Percent rule, one can derive predictions about the percentage of Election Day eligible voters who would have been deterred by lines if HB 589 had been effect during the 2012 general election in North Carolina. Using the scenarios for waiting times discussed earlier, I generated predictions for the turnout effects of long lines, as set forth in Exhibit G.

Exhibit G. Scenarios combining queuing predictions and the Two Percent Rule.[23]

% Increase in ED Voters	Rate (l)	Exp[Wait]	% Deterred	# Deterred Voters
0%	100.0	13.0	0.4%	16,629
2%	102.0	27.0	0.9%	34,591
14%	114.0	37.1	1.2%	47,567
52%	152.0	179.5	6.0%	229,947

39.   As Exhibit G indicates, any further increase in average waiting times would have translated into thousands more deterred voters. For example, a 2% increase in the number of voters arriving on Election Day would mean waiting times of 27 minutes, which would in turn translate into a 0.9% decline in turnout, or 34,591 voters. Given that I estimate that 16,629 voters were already deterred from voting because of lines on Election Day in North Carolina in 2012, this low-end scenario would represent a net of increase of almost 18,000 more voters deterred due to lines. A 14% increase in voters arriving on Election Day would lead to waiting times of 37 minutes, and a 1.2% decline in turnout, or a total of 47,567 deterred voters. The worst-case scenario of 180 minute waiting times would translate into a decline in turnout of 6%, or 230,572 voters. In any scenario, a reduction of early voting days would have resulted in thousands more deterred voters.

40.   As noted above, these calculations may underestimate the effect that eliminating 7 days of early voting would have on line length and waiting times. This may, in turn, lead to an underestimate of the number of voters who would have been deterred from voting. Moreover,

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[23] As noted, the baseline number from which to calculate the percentage of deterred voters is the number of registered voters, minus those voters who have already voted prior to Election Day. For 2012 in North Carolina, I calculate that number as 3,842,873 voters, which I arrived at by taking the number of registered voters for 2012 as reported by the State Board of Elections, *see* http://www.ncsbe.gov/ncsbe/voter-turnout, from which I subtracted the number of voters casting ballots by various means (e.g., in-person, via email, etc.) prior to Election Day, as reported by the State Board of Elections on its official ftp site, *see*: ftp://www.app.sboe.state.nc.us/ENRS/historystats11xx06xx2012.zip.

although not all voters from the now-eliminated days would have tried to vote on Election Day, the shifting of voters *within* the early voting period may have had additional downstream effects on Election Day. That is, eliminating 7 days of early voting would likely have affected not only the lines on Election Day, but possibly could have resulted in longer lines during the remaining days of the early voting period. Longer lines during the remaining early voting period could have, in turn, deterred some of the voters who cast ballots on those days, while shifting others to Election Day. The result would possibly have been additional congestion, longer lines, and more deterred voters on Election Day. In sum, there may be reasons to think that the above calculations actually underestimate the suppressive effects that eliminating 7 days of early voting would have on voting.

VIII. CONCLUSIONS

41.   I render the following preliminary opinions within a reasonable degree of professional certainty. I retain the right to amend or supplement my opinions if additional information or materials become available.

42.   Predicting turnout in a future election, as well as the behavior and preference of voters, is difficult. Also, election officials could take measures to increase the numbers of machines available and/or shorten the ballot lengths in ways that are difficult to predict at this time. Yet, if factors such as the number of voting machines and the time it takes to vote are held constant, then queuing theory, empirical evidence, and common sense all tell us that, if more voters arrive during a given time period, waiting times to vote will increase.

43.   Here, over 899,000 voters (or 19.8% of all voters in 2012) voted on days which are no longer permitted. By examining many scenarios in which some of these voters would have tried to vote on Election Day, and using established modeling approaches, I conclude that eliminating 7 days of early voting before the 2012 election would have caused waiting times to

vote to increase significantly. Even a modest increase of voters trying to vote on Election Day of 2% (*i.e.*, 34,397 voters, or just 3.8% of the approximately 899,083 voters from the now-eliminated days), would result in lines approaching an average waiting time of 27 minutes, or more than double what was actually reported in 2012 in North Carolina. In the worst-case scenario, we would have seen 3-hour waits, the same type of lines actually experienced in 2004 in Franklin, Ohio and in 2012 in central Florida. These are conservative estimates, for a number of reasons, including the fact that these estimates do not take into account the effect of HB 589 on waiting times to vote during the early voting period.

44.   Of course, 3 hours is a worst-case scenario; but I conclude that, if the lines had reached even a fraction of that length, thousands of the Election Day eligible voting population would likely have been deterred from voting. Even the low-end estimate of 27-minute average waiting times to vote, would translate into 34,591 voters being deterred, or more than double the number of voters whom I estimate were actually deterred due to line length in 2012. Some of these people might likely have personal or work commitments such that they would have been effectively disenfranchised.

45.   As I[24] and others[25] have noted, one simple way to prevent long lines is to reduce the number of people who show up to vote at the polls on Election Day, and one simple way to do so is by encouraging early voting. Another way to prevent lines is to make casting a ballot quicker, *e.g.*, through allowing straight ticket voting. HB 589 does precisely the opposite, providing fewer opportunities to vote early and increasing the average time that it will take to

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[24] Yang, Fry, Kelton, & Allen (in press), *supra* note 6.

[25] Levitt, J. (2013). "Fixing That": Lines at the Polling Place. *Journal of Law and Politics* 28(4): 474-75; Belenky, A. S. & Larson, R. C. (2006). To Queue or Not to Queue? In a U.S. Presidential Election, that should NOT Be a Question! *Operations Research / Management Sciences Today* 33(3): 30-35.

cast a ballot. Looking ahead, I believe that HB 589 creates the potential for causing substantially longer waiting times to vote, and for deterring a significant number of eligible voters who would have otherwise turned out to vote on Election Day.

I declare under penalty of perjury under the laws of the United States that the forgoing is true and correct to the best of my knowledge.

DATED this 11th Day of April, 2014

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Theodore Allen, PhD

Appendix A – Curriculum Vitae

THEODORE T. ALLEN

Department of Industrial, Welding & Systems Engineering
The Ohio State University, 210 Baker Systems, 1971 Neil Ave., Columbus, OH 43210
(614) 292-1793 www-iwse.eng.ohio-state.edu/~facultyp/allen.htm

Education

PRINCETON UNIVERSITY, B.A. 1991, Physics (Honors), certificate in Engineering Physics

UCLA, M.S. 1992, Physics (Solid State)

THE UNIVERSITY OF MICHIGAN, M.S. 1994, Ph.D.1997, Industrial & Operations Engineering

Appointments

THE OHIO STATE UNIVERSITY (OSU), Industrial, Welding & Systems, Columbus, OH
8/04-present Associate Professor
8/97-8/04 Assistant Professor
8/96-8/97 Instructor

FORD MOTOR COMPANY, Climate Control Operations, Advanced Engineering
5/94 –8/96 Advanced engineer to heating, ventilating, and cooling including fans and cowls

Publications

(i) Selected Five Relevant to Proposed Research
1. Yang, M., M. Fry, D. Kelton, and T. T. Allen (to appear), "Improving Voting Systems through Service-Operations Management," *Production and Operations Management*.
2. Yang, M., T. T. Allen, M. Fry, and D. Kelton (2013), "The Call for Equity: Simulation-Optimization Models to Minimize the Range of Waiting Times," *IIE Transactions*, 45, 1–15.
3. Li, J., T. T. Allen, and K. Akab (2013), "Could Simulation Optimization Have Prevented 2012 Central Florida Election Lines," *Proceedings of the 2013 Winter Simulation Conference*, R. Pasupathy, S.-H. Kim, A. Tolk, R. Hill, and M. E. Kuhl, eds.
4. Afful-Dadzie, A. and T. T. Allen (2013), "Sufficiency Model-Action Clarification for Simulation Optimization Applied to an Election System," *Proceedings of the 2013 Winter Simulation Conference*, R. Pasupathy, S.-H. Kim, A. Tolk, R. Hill, and M. E. Kuhl, eds.
5. Allen, T. T. (2011), *Introduction to Discrete Event Simulation Theory with Applications: Voting Systems, Health Care, Military, and Manufacturing*, Springer Verlag: London.

(ii) Selected Additional Five Significant Publications
1. Afful-Dadzie, A. and T. T. Allen (to appear), "Data-Driven Cyber Vulnerability Maintenance Policies," *Journal of Quality Technology*.
2. Allen, T. T. and R. Rajagopalan (2011), "Response-Probability Model Analysis Plots," *Journal of Quality Technology*, 43, 3, 224-235.
3. Allen, T. T. (2010), *Introduction to Engineering Statistics and Lean Sigma: Statistical Quality Control and Design of Experiments and Systems*, 2nd ed., Springer Verlag: London.

4. Huang, D., T. T. Allen, W. Notz, and N. Zheng (2006), "Global Optimization of Stochastic Black-Box Systems via Sequential Kriging Meta-Models," *Journal of Global Optimization*, 34 (3), 427-440.
5. Allen, T. T. and M. Bernshteyn (2003), "Supersaturated Designs that Maximize the Probability of Finding the Active Factors," *Technometrics*, 45 (1), 1-8.

Synergistic Activities

1. **Innovations in Teaching and Training:** (1) Developed the syllabus, course materials, and taught over 2,500 engineering students applied statistics in courses including simulation and design of experiments, (2) developed second edition textbook with all sales (in print and web) exceeding 10,000.
2. **Professional Service:** INFORMS Prize Committee. INFORMS Social Media Analytics Newsletter Editor. Lead team to implement theory-based machine allocation in Franklin County, Ohio in 2008 and 2010. Lead technical team to implement theory-based allocation in Cuyahoga County, Ohio in 2008. The allocations helped to reduce waiting times and increase equity in the presidential election.
3. **Founder and sole proprietor of Sagata Ltd.:** Software and consulting company.
4. **Additional Service:** Expert witness in New Mexico and North Carolina.
5. **Editorial Positions:** Associate Editor *Quality Approaches in Higher Education*, Editorial board of *IJISE, IJSSE,* and *IJEDPO*. 2009 reviewer for: *Bioinformatics*, *IIE Transactions*, *Technometrics*, *JQT*, *IJQSR*, *QE*, and *CDSA*.

Collaborations and Other Affiliations

1. **Ph.D. Advisors:** Gary Herrin (University of Michigan, chair), Vijay Nair (University of Michigan), C. J. F. Wu (Georgia Tech University)
2. **Collaborators:** David Farson (The Ohio State University), Nicholas Hall (The Ohio State University), Mario Lauria (Telethon Institute of Genetics and Medicine), William Notz (The Ohio State University), Fritz Scheuren (NORC), David Woods (The Ohio State University)
3. **Ph.D. Students (chair or primary chair):** Afful-Dadzie, Anthony (2012), Lee, Soo Ho (2012), Hui Xiong (2011), Ravishankar Rajagopalan (2009), Taslim, Cenny (2008), Zheng, Ning (2008), Tseng, Shih-Hsien (2008), Schenk, Jason (2008), Ferhatosmanoglu, Nilgun (2007), Brady, James (2005), Huang, Deng (2005), Chantarat, Navara (2003), Ittiwattana, Waraphorn (2002), Bernshteyn, Mikhail (2001), Ribardo, Charles (2000), Yu, Liyang (2000)

Honors and Awards

1. Winner *2013 Quality Approaches In Higher Education* Best Paper Prize
2. Fellow of the *American Society of Quality* (ASQ)
3. Charles E. MacQuigg Student Award for Outstanding Teaching (2001, OSU)
4. Alpha Pi Mu Outstanding Faculty Awards (2013, 2011, 2010, 2002, and 2001 only a single faculty member gets this each year as voted by the graduating seniors)
5. 1 of 100 "outstanding engineers from engineering and industry" selected to attend the 2010 Frontiers of Engineering Education Symposium by the National Academy of Engineering
6. Interviewed on NYT, CNN, CNNi, and in *Time* for voting machine allocation expertise

Appendix B – Documents Considered

1.   General Assembly of North Carolina, Session Law 2013-381 (House Bill 589).

2.   North Carolina Board of Elections data from http://www.ncsbe.gov/ncsbe/voter-turnout.

3.   Summary of Key Changes to Voting Laws in HB589.docx (word document summarizing major provisions of HB 589).

4.   Raw_Vote_Totals_OneStope_2010-2012.xlsx (spreadsheet with counts by day of total accepted ballots, data from North Carolina State Board of Elections official FTP archive; data available at ftp://www.app.sboe.state.nc.us/ENRS/absentee11xx06xx2012.zip; ftp://www.app.sboe.state.nc.us/ENRS/historystats11xx06xx2012.zip; ftp://www.app.sboe.state.nc.us/ENRS/absentee11xx02xx2010.zip; and ftp://www.app.sboe.state.nc.us/ENRS/historystats11xx02xx2010.zip.

5.   Voter history text file from North Carolina State Board of Elections official FTP archive, Historystats11xx062012.txt, available at ftp://www.app.sboe.state.nc.us/ENRS/.

6.   Deposition Transcript of Charlie Collicut, Guilford County Board of Elections, *Dickson v. Rucho*, Nos. 11 CVS 16896, 11 CVS 16940 (N.C. Sup. Ct., Wake County), March 15, 2012. Including Exhibits 34 (summary data from 2008); 36 (2008 early voting schedules and location); and 37 (2010 early voting schedules and location).

7.   Sample ballots from 2012 in Cumberland, Forsyth, Guilford, and Mecklenburg Counties in North Carolina.

8.   Poll closing data in the form of sampled printouts from DRE machines in Cumberland County, North Carolina in 2012.